New Origin of a Bilinear Mass Matrix Form

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Abstract

The charged lepton mass formula can be explained when the masses are propotinal to the squared vacuum expectation values (VEVs) of scalar fields. We introduce U(3) flavor symmetry and its nonet scalar field Φ, whose VEV structure plays an essential role for generating the fermion mass spectrum. We can naturally obtain bilinear form of the Yukawa coupling $Y_{ij} \propto \sum_k \langle \Phi_{ik} \rangle \langle \Phi_{kj} \rangle$ without the non-renormalizable interactions, when the flavor symmetry is broken only through the Yukawa coupling and tadpole terms. We also speculate the possible VEV structure of $\langle \Phi \rangle$.

The observed mass spectra of the quarks and leptons might provide an important clue for the underlying theory. For the charged lepton sector, we know the following empirical mass relation[1, 2],

$$m_e + m_\mu + m_\tau = \frac{2}{3}(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2, \tag{1}$$

which can give a remarkable prediction $m_\tau = 1776.97$ MeV from the observed values of m_e and m_μ. (The observed value is $m_\tau^{obs} = 1776.99^{+0.29}_{-0.26}$ MeV [3].) This mass relation seems to give remarkable hints for the origin of the mass spectrum. In order to get the mass relation (1), an interesting idea was proposed in Ref.[2]: the mass spectrum originates not in the structure of the Yukawa coupling constants Y_{ij} but of the vacuum expectation values (VEVs) v_is of scalars ϕ_is as

$$v_1^2 + v_2^2 + v_3^2 = \frac{2}{3}(v_1 + v_2 + v_3)^2. \tag{2}$$

Here we encounter following two questions.

(i) How can we obtain the VEV relation (2) naturally?

(ii) How to build a model in which m_{ei} has a bilinear form

$$m_{ei} \propto v_i^2, \tag{3}$$

naturally?

The first question seems to be related to a permutation symmetry of S_3[4] or higher symmetries which contain S_3. The second question can be solved by the seesaw-type mass generation mechanism for the charged fermions [5]. However, in the seesaw-type model, we must identify the scalar ϕ as the three Higgs doublets with $\mathcal{O}(10^2)$ GeV VEVs, which may induce the unwanted large flavor changing neutral current (FCNC) [7]. On the other hand, ϕs are not Higgs doublets

in the Froggatt-Nielsen-type model[6] so that the FCNC problem might be avoided. However, it should be emphasized that the bilinear form is just an assumption in the Froggatt-Nielsen-type model.

The purpose of this paper is to propose a new mechanism which induces the bilinear form $m_{ei} \propto v_i^2$ in the framework of a SUSY scenario. The SUSY model which leads to the VEV relation (2) and the bilinear form has been firstly proposed by Ma [8], where four Higgs fields $(\eta_i, \xi_i, \zeta_i, \psi_i)$ were introduced[1]. The bilinear structure $m_{ei} \propto v_i^2$ has been realized via $m_{ei} \propto \langle \eta_i^0 \rangle \propto \langle \zeta_i^0 \rangle \langle \sigma_i \rangle \propto \langle \sigma_i \rangle^2$, where $\langle \sigma_i \rangle$ satisfies the VEV relation (2). This model is well organized but there are too many Higgs doublets. In this paper, we will try to construct a new model which naturally induces the bilinear form of $Y_i j \propto \sum_k \langle \Phi_{ik} \rangle \langle \Phi_{kj} \rangle$ in the different way from Refs.[8] and [9]. We will introduce only one SU(2)$_L$-singlet superfield Φ which plays a role of giving the VEV relation (2) in addition to the conventional set of Higgs doubles, H_d and H_u, which give the masses of the charged leptons (and also the down-quarks) and the neutrinos (and also the up-quarks), respectively.

Under an flavor symmetry, the leptons L_i and E_i are transformed as

$$L = U_X L', \quad E = U_X E', \tag{4}$$

where L_i and E_i are the left-handed SU(2)$_L$ doublets and the SU(2)$_L$ singlets, respectively. We do not specify whether the transformation U_X is continuous or discrete. In the conventional model, the Yukawa interaction of the charged lepton sector is given by

$$W_Y = \sum_{i,j} Y_{ij} L_i H_d E_j = \mathrm{Tr}[Y(E H_d L)]. \tag{5}$$

The Yukawa coupling constants Y_{ij} are strictly constrained by the symmetry under U_X, or the symmetry is badly broken by the Yukawa interaction (5). We would like to consider the structure-less Yukawa coupling, and the mass spectrum originates not in the Yukawa coupling constants Y but in the VEV of scalars. In order for the Yukawa interactions to be invariant under the transformation U_X, we introduce a nonet scalar Φ which transforms as

$$\Phi = U_X \Phi' U_X^\dagger. \tag{6}$$

When the flavor symmetry is U(3), the scalar Φ is regarded as a nonet. A prototype model with a U(3) nonet scalar is found in Ref.[2], and a more realistic U(3) nonet model is proposed in Ref.[9]. The general form of W_Φ is given by

$$W_\Phi = m_1 \mathrm{Tr}[\Phi\Phi] + m_2 (\mathrm{Tr}[\Phi])^2 + \lambda_1 \mathrm{Tr}[\Phi\Phi\Phi] + \lambda_2 \mathrm{Tr}[\Phi\Phi]\mathrm{Tr}[\Phi] + \lambda_3 (\mathrm{Tr}[\Phi])^3. \tag{7}$$

A suitable choice of the parameters might give non-zero magnitude of $\langle \Phi \rangle$, and an effective Yukawa interaction can be induced from

$$y \frac{1}{M} \mathrm{Tr}[\Phi(E H_d L)], \tag{8}$$

[1] η_i, ξ_i, ζ_i, and ψ_i are SU(2)$_L$-doublet Higgs fields, and η_i has the Yukawa interaction $f \eta_i L_i E_i$.

which is invariant under the transformation of U_X. This is a Froggatt-Nielsen-type model proposed in Ref.[9]. The interaction (8) is a higher dimensional term which is accompanied with an energy scale M of the effective theory, and the bilinear form is not derived. We will seek for another mechanism which can give $m_{ei} \propto v_i^2$ through the renormalizable interactions.

Conventional models have considered exact unbroken flavor symmetries at the beginning, which are spontaneously broken later. In this paper we take a different setup where the superpotential W has explicit (U_X) symmetry breaking terms, which are common in Yukawa interaction (4) and a tadpole terms $\mathrm{Tr}[Y\Phi]$ as

$$W = W_\Phi - \mu^2 \mathrm{Tr}[Y\Phi] + W_Y. \tag{9}$$

This shows

$$\frac{\partial W}{\partial \Phi} = 0 = \frac{\partial W_\Phi}{\partial \Phi} - \mu^2 Y = 3\lambda_1 \Phi\Phi + f_1(\Phi)\Phi + f_0(\Phi)\mathbf{1} - \mu^2 Y, \tag{10}$$

where

$$f_1(\Phi) = 2(m_1 + \lambda_2 \mathrm{Tr}[\Phi]), \tag{11}$$

$$f_0(\Phi) = 2m_2 \mathrm{Tr}[\Phi] + \lambda_2 \mathrm{Tr}[\Phi\Phi] + 3\lambda_3 (\mathrm{Tr}[\Phi])^2, \tag{12}$$

and $\mathbf{1}$ is a 3×3 unit matrix. Now we put an ansatz that our vacuum is given by the solution of Eq.(10) as

$$3\lambda_1 \Phi\Phi - \mu^2 Y = 0, \tag{13}$$

and

$$f_1(\Phi)\Phi + f_0(\Phi)\mathbf{1} = 0. \tag{14}$$

The requirement (13) realizes the bilinear relation of our goal as

$$Y_{ij} = \frac{3\lambda_1}{\mu^2} \sum_k \langle \Phi_{ik} \rangle \langle \Phi_{kj} \rangle. \tag{15}$$

For the existence of non-zero and non-degenerate eigenvalues of v_i, Eq.(14) requires $f_1 = 0$ and $f_0 = 0$, i.e.

$$\mathrm{Tr}[\Phi] = -\frac{m_1}{\lambda_2}, \tag{16}$$

and

$$2m_2 \mathrm{Tr}[\Phi] + \lambda_2 \mathrm{Tr}[\Phi\Phi] + 3\lambda_3 (\mathrm{Tr}[\Phi])^2 = 0. \tag{17}$$

Since the value of $\langle \Phi \rangle$ is of the order of m_1/λ_2, the Yukawa coupling constant Y is of the order of m_1^2/μ^2.

Now let us consider how to obtain the VEV relation (2). When we denote the nonet Φ in terms of the octet $\Phi^{(8)} = \Phi - \frac{1}{3}\mathrm{Tr}[\Phi]$ and the singlet $\Phi^{(1)} = \frac{1}{3}\mathrm{Tr}[\Phi]\mathbf{1}$[2], the term $\mathrm{Tr}[\Phi\Phi\Phi]$ is devided into the following two parts,

$$\mathrm{Tr}[\Phi\Phi\Phi] = \mathrm{Tr}[\Phi^{(8)}\Phi^{(8)}\Phi^{(8)}] + \mathrm{Tr}[3\Phi^{(1)}\Phi^{(8)}\Phi^{(8)} + \Phi^{(1)}\Phi^{(1)}\Phi^{(1)}], \tag{18}$$

[2]Notice that $\mathrm{Tr}[\Phi^{(8)}] = 0$.

$$\mathrm{Tr}[\Phi^{(8)}\Phi^{(8)}\Phi^{(8)}] = \mathrm{Tr}[\Phi\Phi\Phi] - \mathrm{Tr}[\Phi]\left(\mathrm{Tr}[\Phi\Phi] - \frac{2}{9}(\mathrm{Tr}[\Phi])^2\right), \tag{19}$$

$$\mathrm{Tr}[3\Phi^{(1)}\Phi^{(8)}\Phi^{(8)} + \Phi^{(1)}\Phi^{(1)}\Phi^{(1)}] = \mathrm{Tr}[\Phi]\left(\mathrm{Tr}[\Phi\Phi] - \frac{2}{9}(\mathrm{Tr}[\Phi])^2\right). \tag{20}$$

As shown in Ref.[9], by imposing the Z_2 invariance (Z_2 parities $+1$ and -1 are assigned to the fields $\Phi^{(1)}$ and $\Phi^{(8)}$, respectively), the component $\mathrm{Tr}[\Phi^{(8)}\Phi^{(8)}\Phi^{(8)}]$ with the negative parity is dropped from the term $\mathrm{Tr}[\Phi\Phi\Phi]$ which induces the VEV relation (2). Unfortunately, we cannot apply this Z_2 symmetry to our model because it derives $\lambda_1 = 0$. So we just assume that the cubic term is given by Eq.(19) as[3]

$$W_\Phi = m\mathrm{Tr}[\Phi\Phi] + \lambda\mathrm{Tr}[\Phi^{(8)}\Phi^{(8)}\Phi^{(8)}] \tag{21}$$

in the present stage. Since the cubic term $\mathrm{Tr}[\Phi\Phi\Phi]$ in the expression (19) can be canceled with the tadpole term $-\mu^2\mathrm{Tr}[Y\Phi]$, the remaining terms are essentially identical with the expression (20). Then the assumption (21) gives

$$m_1 = m, \quad m_2 = 0, \quad \lambda_1 = \lambda, \quad \lambda_2 = -\lambda, \quad \lambda_3 = \frac{2}{9}\lambda, \tag{22}$$

which leads to the VEV relation

$$\mathrm{Tr}[\Phi\Phi] = \frac{2}{3}(\mathrm{Tr}[\Phi])^2, \tag{23}$$

with Eq.(17). The relation (23) is the VEV relation (2) on the basis of $\langle\Phi_{ij}\rangle = \delta_{ij}v_i$.

Now, let us discuss the neutrino sector. If the same scalar Φ contributes to the neutrino sector, we cannot explain the observed value [10, 11]

$$R \equiv \frac{\Delta m^2_{solar}}{\Delta m^2_{atm}} = \frac{(7.9^{+0.6}_{-0.5}) \times 10^{-5}\mathrm{eV}^2}{(2.74^{+0.44}_{-0.26}) \times 10^{-3}\mathrm{eV}^2} = (2.9 \pm 0.5) \times 10^{-2}, \tag{24}$$

because this gives too small value of $R \simeq (m_\mu/m_\tau)^2 = 3.4 \times 10^{-3}$ for Dirac neutrinos $m_i^{Dirac} \propto v_i^2 \propto m_{ei}$, and $R \simeq (m_\mu/m_\tau)^4 = 1.2 \times 10^{-5}$ for Majorana neutrinos with $m_{\nu i} \propto (m_i^{Dirac})^2$. So we should consider that the scalar Φ which contributes to the neutrino sector is different from the charged lepton sector (we will refer the former as Φ_u and the latter as Φ_d). We would like to consider that the essential structure of the superpotential $W(\Phi_u)$ is the same as $W(\Phi_d)$ with the relation (23) for $\langle\Phi_u\rangle$. Here, let us define a useful notation of dimensionless parameters z_i which is defined by $v_i = vz_i$, where $v = \sqrt{v_1^2 + v_2^2 + v_3^2}$. Then, the values z_is satisfy the relation $z_1^2 + z_2^2 + x_3^2 = 1 = (2/3)(z_1 + z_2 + z_3)^2$. Remembering that three real solutions x_is of a cubic equation $ax^3 + bx^2 + cx + d = 0$ are expressed by a form $x_i = \alpha + \beta\sin(\theta + (2/3)(i-1)\pi)$ $(i = 1, 2, 3)$, the parameters z_is can be expressed by

$$\begin{aligned} .z_1 &= \tfrac{1}{\sqrt{6}} - \tfrac{1}{\sqrt{3}}\sin\theta, \\ z_2 &= \tfrac{1}{\sqrt{6}} - \tfrac{1}{\sqrt{3}}\sin\left(\theta + \tfrac{2}{3}\pi\right), \\ z_3 &= \tfrac{1}{\sqrt{6}} - \tfrac{1}{\sqrt{3}}\sin\left(\theta + \tfrac{4}{3}\pi\right), \end{aligned} \tag{25}$$

[3] The form (21) is only a phenomenological assumption.

since v_i are eigenvalues of the 3×3 matrix of $\langle \Phi \rangle$[4]. Thus the ratio of (24) is written as

$$R_n = \frac{z_2^n - z_1^n}{z_3^n - z_2^n}. \tag{26}$$

If neutrino masses are Dirac type without a seesaw mechanism, the observed ratio (24) is given by Eq.(26) with $n = 4$. On the other hand, if neutrino masses are Majorana type which are generated by a seesaw mechanism with $M_R \propto \mathbf{1}$, the ratio is given by Eq.(26) with $n = 8$. They suggest

$$\theta_\nu = 57.0^\circ \pm 1.4^\circ, \tag{27}$$

for $R_4 = 0.029 \pm 0.005$ and

$$\theta_\nu = 72.5^\circ \pm 0.8^\circ, \tag{28}$$

for $R_8 = 0.029 \pm 0.005$[5]. As for the charged lepton sector, the observed charged lepton masses (m_e, m_μ, m_τ) suggest

$$\theta_e = 42.7324^\circ, \tag{29}$$

which give $z_1 = 0.016473$, $z_2 = 0.236869$ and $z_3 = 0.971402$. It is interesting that the value (28) satisfies $\theta_\nu - \theta_e \simeq 30^\circ$.

So far, we have not discussed the neutrino mixing. Notice that the results (16) – (17) (and also (23)) are satisfied independently of the flavor basis. The Yukawa coupling constants Y_ν and Y_e are related to the VEV relations $\langle \Phi_f \rangle$ $(f = u, d)$ as

$$Y_\nu = \frac{3\lambda_u}{\mu_u^2} \langle \Phi_u \rangle^2, \quad Y_e = \frac{3\lambda_d}{\mu_d^2} \langle \Phi_d \rangle^2. \tag{30}$$

So if we fix the flavor basis of L_i, the basis of $(Y_\nu)_{ij}$ and $(Y_e)_{ij}$ are also fixed. For an example, if we choose the flavor basis in which Y_e is diagonal ($\langle \Phi_d \rangle$ is diagonal), the matrix Y_ν ($\langle \Phi_u \rangle$) is not diagonal on this basis in general. So far we can only know the eigenvalues of $\langle \Phi_f \rangle$ and cannot know the explicit form of the matrix $\langle \Phi_u \rangle$. In order to fix the flavor mismatch between Y_ν and Y_e, we try to introduce an additional term $\varepsilon \mathrm{Tr}[B_f \Phi_f]$ in the superpotential from the practical point of view as

$$W_f = W_{\Phi_f} - \mu_f^2 \mathrm{Tr}[Y_f \Phi_f] + W_{Y_f} + \varepsilon \mathrm{Tr}[B_f \Phi_f], \tag{31}$$

where B_f are not fields but numerical matrices. We assume that the basis where the VEV matrix $\langle \Phi_f \rangle$ becomes diagonal is fixed by the condition

$$\mathrm{Tr}[B_f \Phi_f] = \mathrm{Tr}[U_f^\dagger B_f U_f \widetilde{\Phi}_f] = 0, \tag{32}$$

where

$$\widetilde{\Phi}_f \equiv \mathrm{diag}(v_{f1}, v_{f2}, v_{f3}) = U_f^\dagger \Phi_f U_f. \tag{33}$$

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[4] The factor $1/\sqrt{6}$ is coming from the normalization of $(z_1 + z_2 + z_3)^2 = 3/2$.

[5] Here we chose the case $z_1^2 < z_2^2 \ll z_3^2$. Since we have not fixed the neutrino mixing matrix so far, we can also choose another solutions of θ_ν by the replacement of $\theta \to 60^\circ - \theta$ which corresponds to the case $z_2^2 < z_1^2 \ll z_3^2$.

Since the matrices B_f have been introduced only for the purpose to fix the flavor basis for the concerned Yukawa interaction, we can take $\varepsilon \to 0$ in the final results. For an example, let us examine the case of [9], where the flavor symmetry is U(3) and it breaks to S_4. The nonet scalar Φ_d is expected to be broken to $\mathbf{1}+\mathbf{2}+\mathbf{3}+\mathbf{3}'$ of S_4 and the components of $\mathbf{1}+\mathbf{2}$ generate the charged lepton masses. This splitting between $\mathbf{1}+\mathbf{2}$ and $\mathbf{3}+\mathbf{3}'$ is realized by a matrix

$$B_e = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \tag{34}$$

because the components Φ_{ij} $(i \neq j)$ denote $\mathbf{3}+\mathbf{3}'$ of S_4 in the nonet expression of Φ, and the components $\Phi_{11} = \frac{1}{\sqrt{3}}\Phi_\sigma + \frac{2}{\sqrt{6}}\Phi_\eta$, $\Phi_{22} = \frac{1}{\sqrt{3}}\Phi_\sigma - \frac{1}{\sqrt{6}}\Phi_\eta - \frac{1}{\sqrt{2}}\Phi_\pi$ and $\Phi_{33} = \frac{1}{\sqrt{3}}\Phi_\sigma - \frac{1}{\sqrt{6}}\Phi_\eta + \frac{1}{\sqrt{2}}\Phi_\pi$ denote a singlet Φ_σ and a doublet (Φ_π, Φ_η) of S_4. In this case, the trace $\mathrm{Tr}[B_e\tilde{\Phi}_d]$ is obviously zero with $U_e = \mathbf{1}$. As for the neutrino sector, the splitting between the doublet of S_4 is crucial so we take

$$B_\nu = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & +1 \end{pmatrix}, \tag{35}$$

which suggests ϕ_π is the component of the doublet (ϕ_π, ϕ_η) of S_4 as in Eq.(33). The matrix B_ν is rotated by

$$U_\nu = U_{TB} \equiv \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{36}$$

as

$$U_{TB}^\dagger B_\nu U_{TB} = \frac{1}{\sqrt{3}} \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & \sqrt{2} \\ -1 & \sqrt{2} & 0 \end{pmatrix}, \tag{37}$$

where the flavor-basis-fixing term $\mathrm{Tr}[B_\nu\Phi_u] = \mathrm{Tr}[U_\nu^\dagger B_\nu U_\nu \widetilde{\Phi}_u]$ can be set to zero for $U_\nu = U_{TB}$. It means that the Yukawa coupling constant Y_ν is given by $Y_\nu = (3\lambda/\mu^2)U_{TB}(\widetilde{\Phi}_u)^2 U_{TB}^\dagger$ on the basis where Y_e is diagonal. This suggests the neutrino mixing matrix is given by "tri/bi-maximal mixing" $U_\nu = U_{TB}$ [14]. Notice that this does not mean we have derived the tri/bi-maximal mixing in our model, since the mixing form is due to the ad hoc choice of (35). The ansatz (32) is only a trial, but the introduction of a flavor-basis-fixing term seems to be an interesting candidate to complete our scenario.

In conclusion, we have examined the idea that the fermion mass spectrum originates not in the structure of the Yukawa coupling but in the VEV structure. We have proposed a new

mechanism which gives a bilinear form of $m_i \propto v_i^2$ without introducing higher dimensional interactions as in the Froggatt-Nielsen model. We have applied this mechanism to the charged lepton mass relation (2) at first. For the derivation of $Y \propto \langle\Phi\rangle^2$, it has been essential that the flavor symmetry of the superpotential $W_\Phi(\Phi)$ is broken only by the tadpole term $\mu^2\mathrm{Tr}[Y\Phi]$, where $\partial W/\partial\Phi = 0$ has derived $Y \propto \langle\Phi\rangle^2$. Notice that the bilinear form (15) is not a unique solution (vacuum), and there are other solutions (vacuums) in the general form of

$$Y = \frac{1}{\mu^2}\left\{3\lambda_1\Phi\Phi + f_1(\Phi)\Phi + f_0(\Phi)\mathbf{1}\right\}. \tag{38}$$

If we take the vacuum where the Yukawa coupling constant Y is only proportional to $\langle\Phi\rangle$, i.e. $\mu^2 Y = 2m_1\langle\Phi\rangle$, we cannot obtain the non-degenerate and non-zero eigenvalues of $\langle\Phi\rangle$. The desirable eigenvalues (non-degenerate and non-zero eigenvalues) exist in the vacuum of $\mu^2 Y = 3\lambda_1\langle\Phi\rangle\langle\Phi\rangle$. When we choose a solution of (13), we obtain $f_1 = f_0 = 0$ as a byproduct in the present scenario. Our purpose of this paper is not the derivation of the formula of (2). We have just assume the form of (21), which induces the VEV relation (2) through the requirement of $f_0(\Phi) = 0$.

We have also applied the same mechanism to the neutrino sector. We have shown one attempt of generating the flavor mixings by introducing the additional interaction. We will seek for more reasonable prescription of generating flavor mixings. In this paper, we have not investigated the quark mass spectra. It is well known that the observed quark masses do not satisfy the relation (23) [(2)] (for example, see Table 1 in Ref.[15]). We will seek for a unified description including quark sectors based on the bilinear mass matrix formulation.

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